①

3.(ii) $\tan\theta + \frac{1}{2}\tan\frac{\theta}{2} + \frac{1}{2^2}\tan\frac{\theta}{2^2} + \frac{1}{2^3}\tan\frac{\theta}{2^3} + \dots$ to n terms

'D₁(4) Maths paper-1st, Gr-A Summation of series Dr. G.D. Singh

we know $t_1 = \tan\theta = \cot\theta - 2\cot 2\theta$ (K)

$t_2 = \tan\frac{\theta}{2} = \cot\frac{\theta}{2} - 2\cot\theta$

$t_3 = \tan\frac{\theta}{2^2} = \cot\frac{\theta}{2^2} - 2\cot\frac{\theta}{2}$

$t_4 = \tan\frac{\theta}{2^3} = \cot\frac{\theta}{2^3} - 2\cot\frac{\theta}{2^2}$

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$t_n = \tan\frac{\theta}{2^{n-1}} = \cot\frac{\theta}{2^{n-1}} - 2\cot\frac{\theta}{2^{n-2}}$

$\therefore \tan\theta = \cot\theta - 2\cot 2\theta$

$\frac{1}{2}\tan\frac{\theta}{2} = \frac{1}{2}\cot\frac{\theta}{2} - 2\times\frac{1}{2}\cot\theta$

$\frac{1}{2^2}\tan\frac{\theta}{2^2} = \frac{1}{2^2}\cot\frac{\theta}{2^2} - 2\times\frac{1}{2^2}\cot\frac{\theta}{2}$

$\frac{1}{2^3}\tan\frac{\theta}{2^3} = \frac{1}{2^3}\cot\frac{\theta}{2^3} - 2\cdot\frac{1}{2^3}\cot\frac{\theta}{2^2}$

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$\frac{1}{2^{n-1}}\tan\frac{\theta}{2^{n-1}} = \frac{1}{2^{n-1}}\cot\frac{\theta}{2^{n-1}} - 2\cdot\frac{1}{2^{n-1}}\cot\frac{\theta}{2^{n-2}}$

Adding we get

$\tan\theta + \frac{1}{2}\tan\frac{\theta}{2} + \frac{1}{2^2}\tan\frac{\theta}{2^2} + \frac{1}{2^3}\tan\frac{\theta}{2^3} + \frac{1}{2^{n-1}}\tan\frac{\theta}{2^{n-1}}$

$= \frac{1}{2^{n-1}}\cot\frac{\theta}{2^{n-1}} - 2\cot 2\theta$

2nd part

when $n \to \infty$

$= \dfrac{\frac{1}{2^{n-1}}}{\tan\frac{\theta}{2^{n-1}}} - 2\cot 2\theta$

(2)

$$= \lim_{n\to\infty} \frac{\frac{\theta}{2^{n-1}} \times \frac{1}{\theta}}{\tan\frac{\theta}{2^{n-1}}} - 2\cot 2\theta \qquad \text{let } y = \frac{\theta}{2^{n-1}}$$

$$= \lim_{y\to 0} \left(\frac{y}{\tan y}\right) \times \frac{1}{\theta} - 2\cot 2\theta \qquad \text{when } n\to\infty,\ y\to 0$$

$$\lim_{\theta\to 0} \frac{\theta}{\tan\theta} = 1$$

$$= 1 \times \frac{1}{\theta} - 2\cot 2\theta = \frac{1}{\theta} - 2\cot 2\theta \text{ Ans}$$

5(i) $\sec\theta \sec 2\theta + \sec 2\theta \sec 3\theta + \sec 3\theta \sec 4\theta + \dots$ to n terms

$$t_r = \sec r\theta \sec (r+1)\theta$$

$$= \frac{1}{\cos r\theta \cos (r+1)\theta} = \frac{1}{\sin\theta} \times \frac{\sin\theta}{\cos r\theta \cdot \cos(r+1)\theta}$$

$$= \frac{1}{\sin\theta} \left\{ \frac{\sin\{(r+1)\theta - r\theta\}}{\cos r\theta \cos(r+1)\theta} \right\}$$

$$= \frac{1}{\sin\theta} \left[\frac{\sin(r+1)\theta \cos r\theta - \cos(r+1)\theta \sin r\theta}{\cos(r+1)\theta \cos r\theta} \right]$$

$$= \frac{1}{\sin\theta} \left\{ \frac{\sin(r+1)\theta \cos r\theta}{\cos(r+1)\theta \cos r\theta} - \frac{\cos(r+1)\theta \sin r\theta}{\cos(r+1)\theta \cos r\theta} \right\}$$

$$t_r = \frac{1}{\sin\theta} \left\{ \tan(r+1)\theta - \tan r\theta \right\}$$

$$t_1 = \frac{1}{\sin\theta} \left\{ \tan 2\theta - \tan\theta \right\}$$

$$t_2 = \frac{1}{\sin\theta} \left\{ \tan 3\theta - \tan 2\theta \right\}$$

$$t_3 = \frac{1}{\sin\theta} \left\{ \tan 4\theta - \tan 3\theta \right\}$$

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$$t_n = \frac{1}{\sin\theta} \left\{ \tan(n+1)\theta - \tan n\theta \right\}$$

$$s = \frac{1}{\sin\theta} \left\{ \tan(n+1)\theta - \tan\theta \right\}$$

(3)

(ii) $\frac{\sin\theta}{\sin 2\theta \sin 3\theta} + \frac{\sin\theta}{\sin 3\theta \sin 4\theta} + \frac{\sin\theta}{\sin 4\theta \cdot \sin 5\theta} + \cdots$ (2, 3, 4, ... = 2+(r-1)1 = 2+r-1 = r+1)

$t_r = \frac{\sin\theta}{\sin(r+1)\theta \cdot \sin(r+2)\theta} = \frac{\sin\{(r+2)\theta - (r+1)\theta\}}{\sin(r+1)\theta \sin(r+2)\theta}$

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6(i) $\frac{1}{\cos\theta - \cos 3\theta} + \frac{1}{\cos\theta - \cos 5\theta} + \frac{1}{\cos\theta - \cos 7\theta} + \cdots$

Here $t_r = \frac{1}{\cos\theta - \cos(2r+1)\theta}$

$= \frac{1}{2\sin\left(\frac{2r+2}{2}\right)\theta \cdot \sin\frac{2r}{2}\theta}$

$= \frac{\sin\theta}{2\sin\theta\left[\sin(r+1)\theta \sin r\theta\right]}$

$= \frac{1}{2\sin\theta}\left[\frac{\sin\{(r+1)\theta - (r\theta)\}}{\sin(r+1)\theta \sin r\theta}\right]$

Rough
3, 5, 7, 9, ..
का r वाँ पद
$= 3 + (r-1)2$
$= 3 + 2r - 2$
$= 2r + 1$

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